KTH ROYAL INSTITUTE OF TECHNOLOGY

Evaluation of Hoffman and Xia plasticity models against bi-axial tension experiments of fiber network materials

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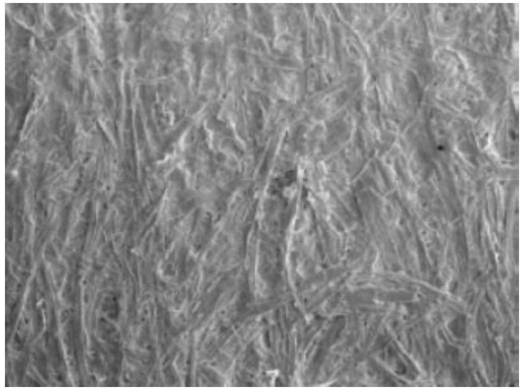
Introduction

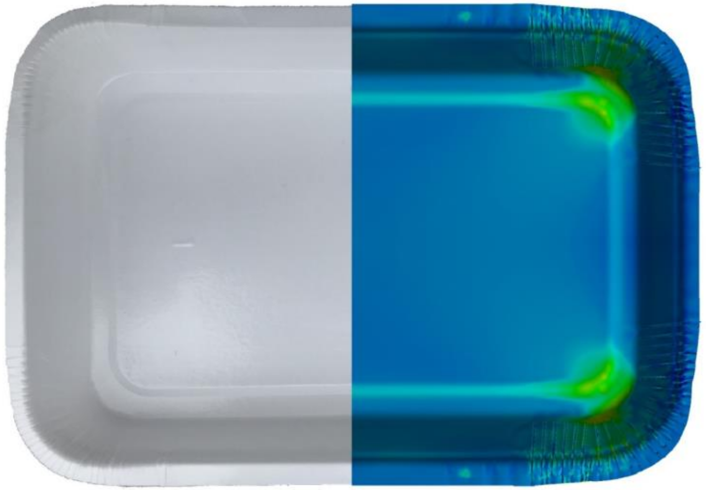
The bio-based materials are of a complex nature

- anisotropy i.e., the material is a directional dependent
- local variations in the properties



Using modeling approach can significantly help during the design phase

- reduce the loop time of design and test
- enhance the end-user experience
- using the computational tool for packaging development
- introducing a virtual twins



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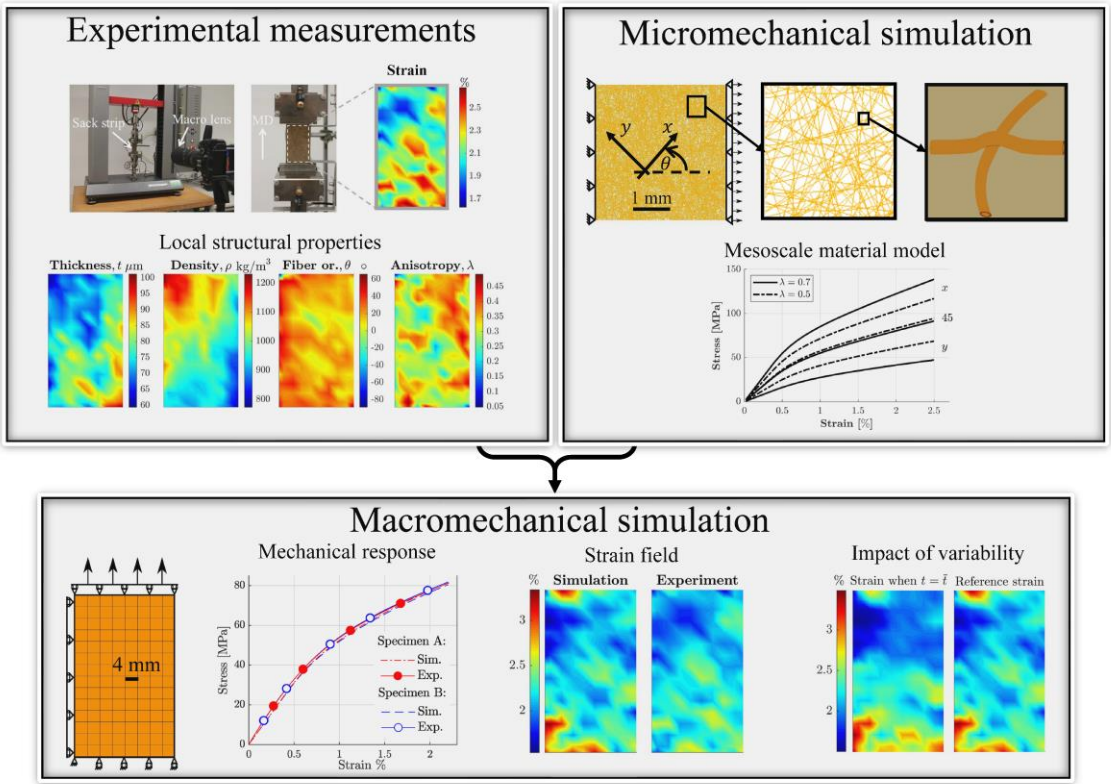
Modelling of bio-based materials

Modeling approaches

- micromechanical simulation tool (more detailed but limited in usage due to the complexity)
- continuum modeling (simplified with broad usage)

Multiscale modeling is used to combine the advantages of both micro and continuum approaches



Alzweighi, M., Mansour, R., Lahti, J., Hirn, U., & Kulachenko, A. (2021). The influence of structural variations on the constitutive response and strain variations in thin fibrous materials. *Acta Materialia*, *203*, 116460.

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Aim and scope

Numerous numbers of continuum models have been developed

There is a lack of comparative studies

Uncertainties regarding the selection of a suitable model step forward

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Methodology

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Approach

Two continuum models are chosen for the benchmark study against biaxial tension experimental results

- Hoffman model
- Xia multi-yield surface

The background of choosing those models are

- Hoffman is of von Mises type
- Xia is of multi yield surface
- both model preset the ability to show anisotropy and asymmetric tension-compression
- these types cover to a large extent most of the continuum approach in bio-based materials

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Material model

Hoffman model

$$f(\boldsymbol{\sigma}, \varepsilon_{\text{eqv}}^{p}) = \frac{1}{2}\boldsymbol{\sigma}^{T}\mathbf{P}\boldsymbol{\sigma} + \mathbf{q}^{T}\boldsymbol{\sigma} - H(\varepsilon_{\text{eqv}}^{p})$$

$\boldsymbol{\sigma}$ stress tensor

$\mathbf{P}$ matrix describes the anisotropy

$\mathbf{q}$ differences in yield stresses in tension and compression

H hardening function

Xia model

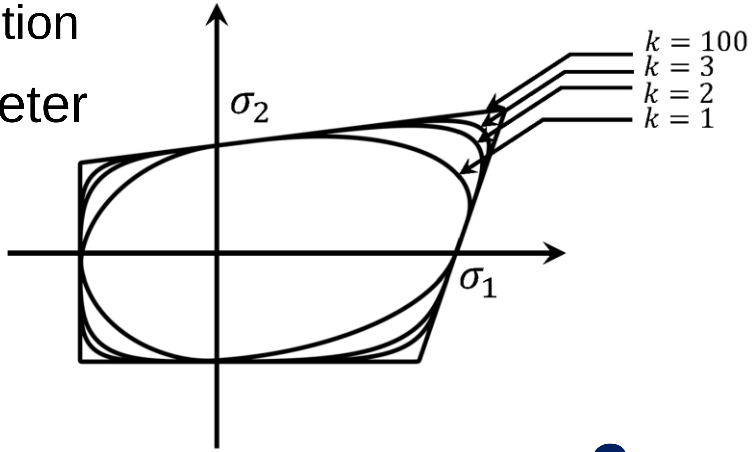
$$f(\boldsymbol{\sigma}, K_{\gamma}) = \sum_{\gamma=1}^{6}\chi_{\gamma}\left(\frac{\boldsymbol{\sigma}^{T}\mathbf{N}_{\gamma}}{K_{\gamma}}\right)^{2k} - 1$$

$$\chi_{\gamma} = \begin{cases} 1 & \boldsymbol{\sigma}^{T}\mathbf{N}_{\gamma} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$\mathbf{N}_{\gamma}$ gradient of the sub-yield surfaces

K_{γ} hardening function

k shape parameter



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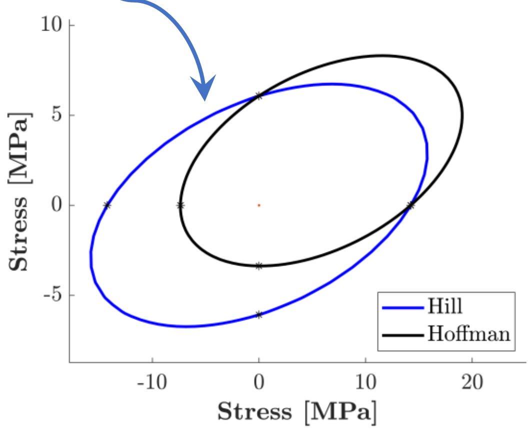
Yield surfaces

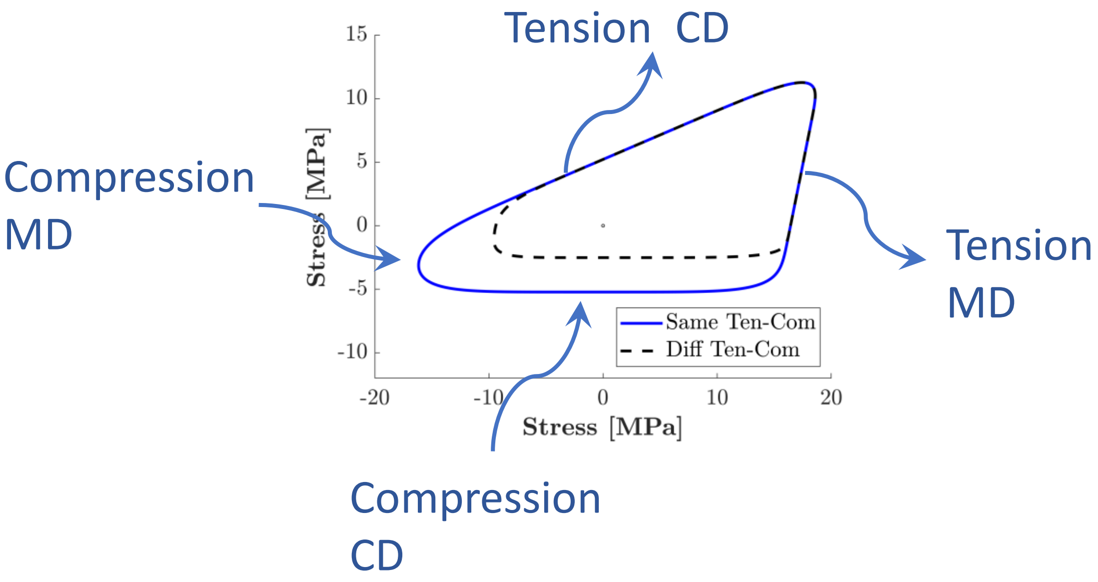
Hoffman model

$$f(\boldsymbol{\sigma}, \varepsilon^{p}_{\mathrm{eqv}}) = \frac{1}{2}\boldsymbol{\sigma}^{T}\mathbf{P}\boldsymbol{\sigma} + \mathbf{q}^{T}\boldsymbol{\sigma} - H(\varepsilon^{p}_{\mathrm{eqv}})$$

0



Xia model



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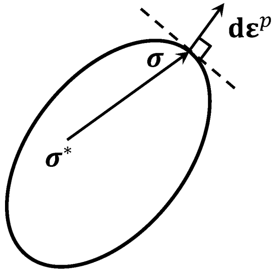
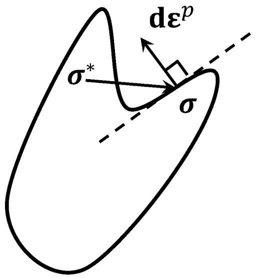
Stability of the models

Drucker's stability postulates

- Normality
- Convexity



convex with normality



concave yield surface with normality

Xia model

$$\mathbf{H}_{\mathrm{Xia}} = \frac{\partial^2 f}{\partial \boldsymbol{\sigma}^2} = \sum_{\gamma=1}^{6} \chi_\gamma 2k(2k-1)\left(\frac{\boldsymbol{\sigma}^T \mathbf{N}_\gamma}{K_\gamma}\right)^{2k-2} \frac{\mathbf{N}_\gamma \mathbf{N}_\gamma^T}{K_\gamma^2} \geq 0$$

unconditionally convex

Hoffman model

$$\mathbf{H}_{\mathrm{Hof}} = \frac{\partial^2 f}{\partial \boldsymbol{\sigma}^2} = \mathbf{P} = \begin{bmatrix} P_{11} & P_{12} & 0 \\ & P_{22} & 0 \\ \mathrm{Sym} & & P_{33} \end{bmatrix}$$

$$\lambda_1 = P_{33} \geq 0$$

$$\lambda_2 = \frac{1}{2}\left(P_{11} + P_{22} + \sqrt{(P_{11} - P_{22})^2 + 4P_{12}^2}\right) \geq 0$$

$$\lambda_3 = \frac{1}{2}\left(P_{11} + P_{22} - \sqrt{(P_{11} - P_{22})^2 + 4P_{12}^2}\right) \geq 0$$

conditional convexity

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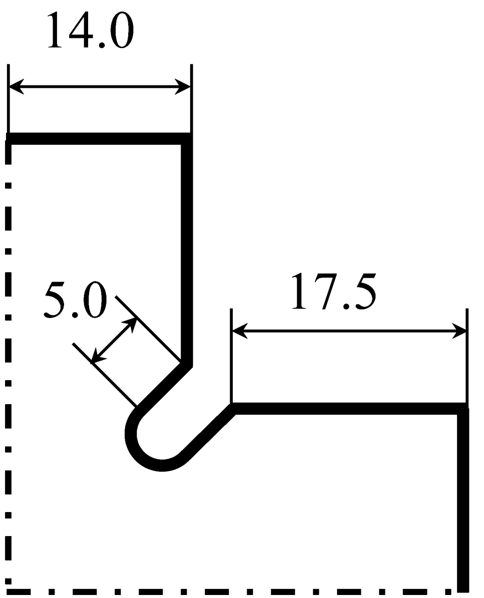
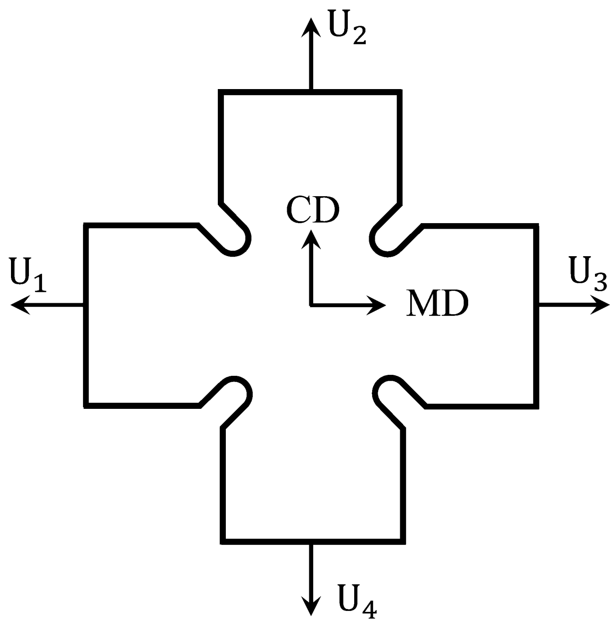
Experimental setup: Geometries

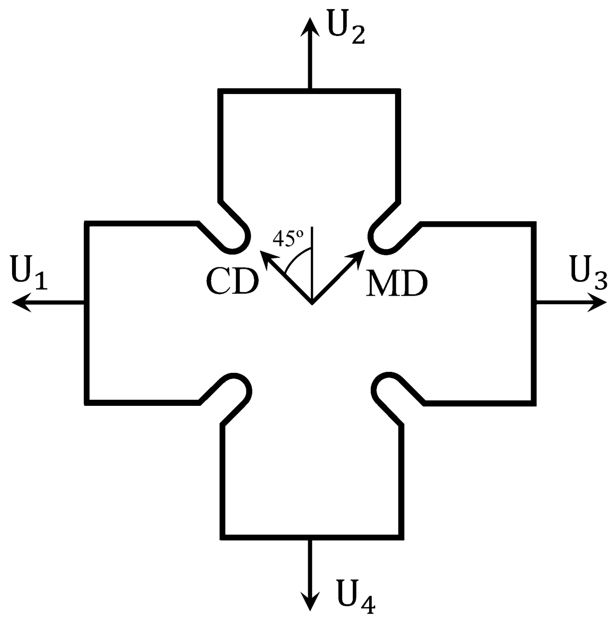
Cruciform specimen with two tests setup



Specimen dimensions (mm)



MD-CD Test setup



45-Rot Test setup

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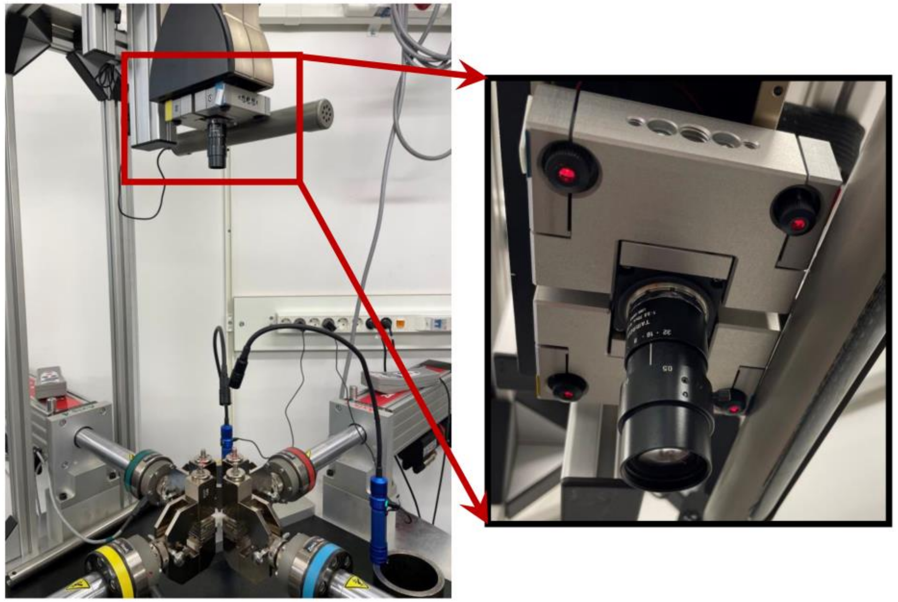
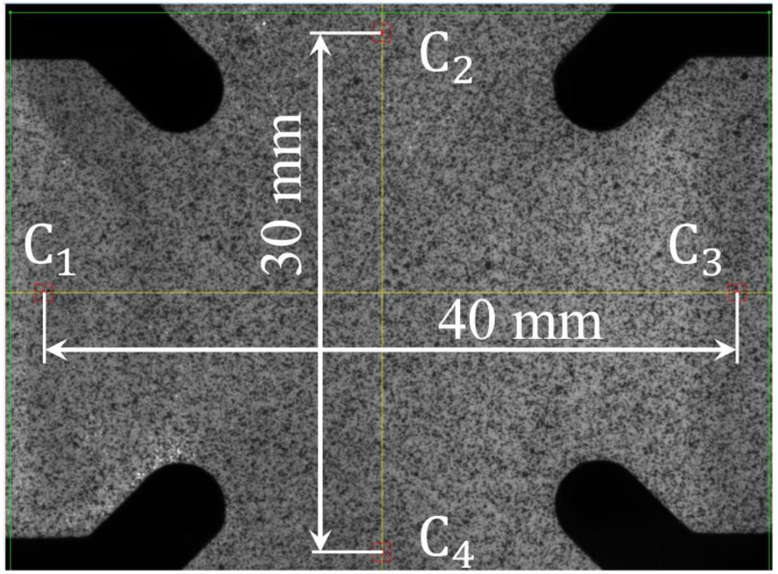
Experimental setup: Testing machine



The optical-extensometer device attached to the bi-axial test machine



The four tracking squares

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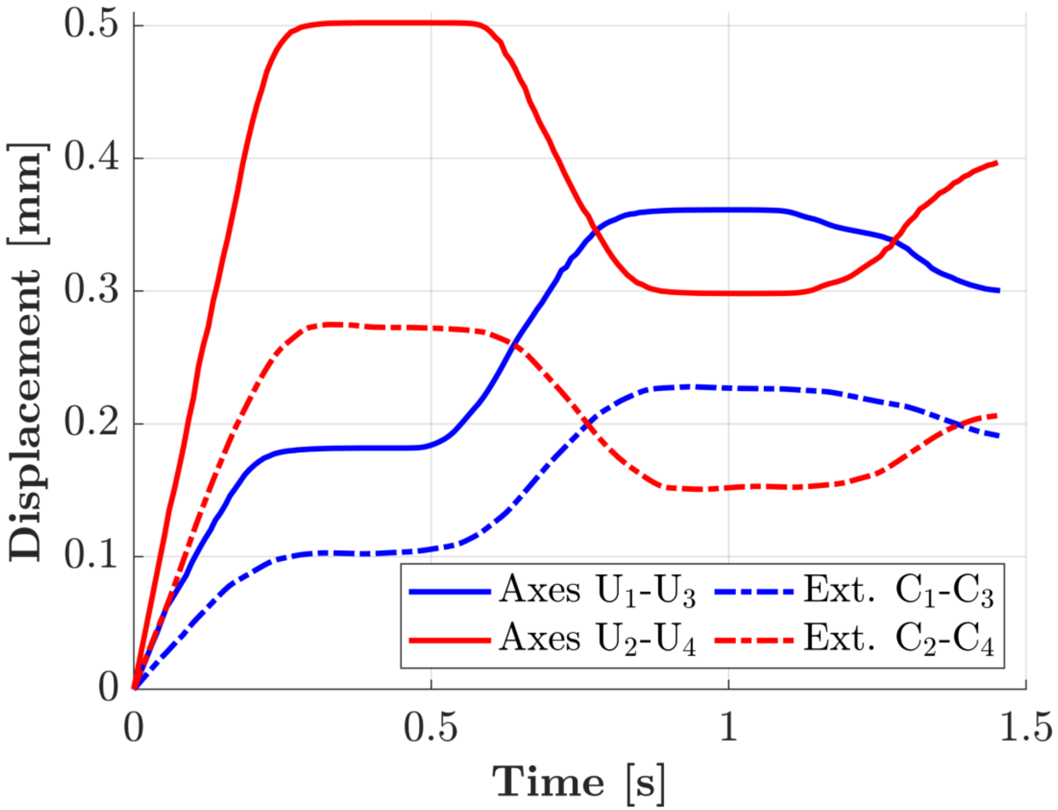
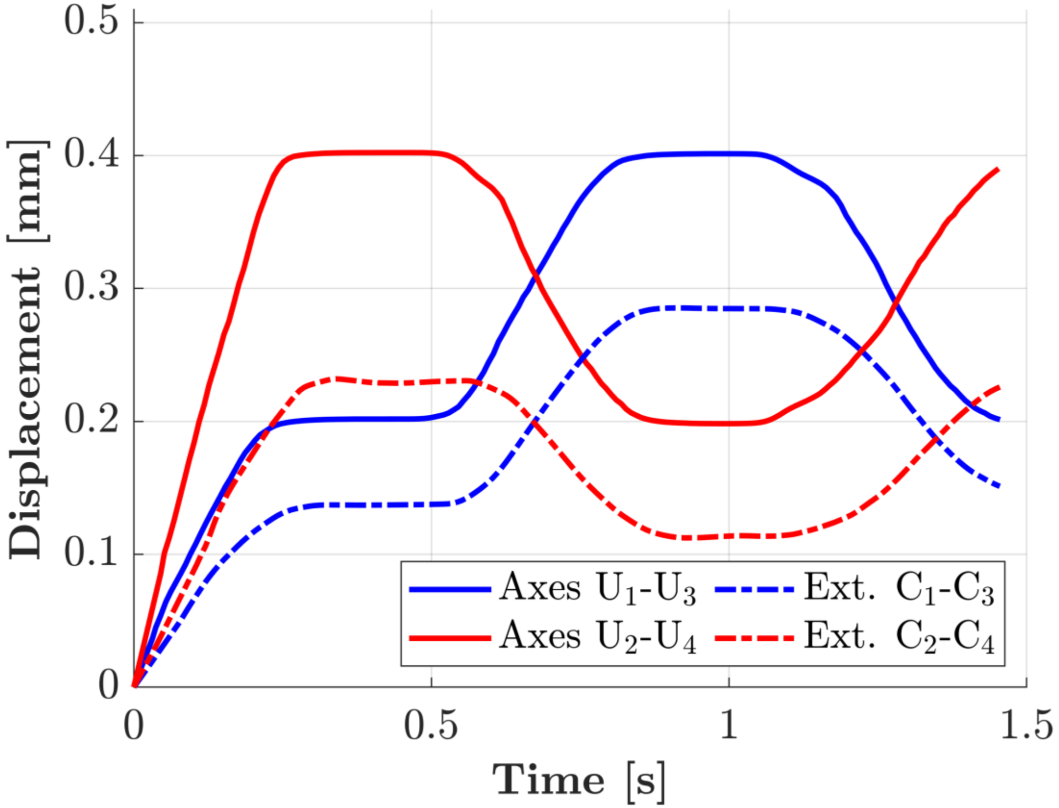
Experimental setup: Time-displacment



MD-CD Test



45-Rot Test

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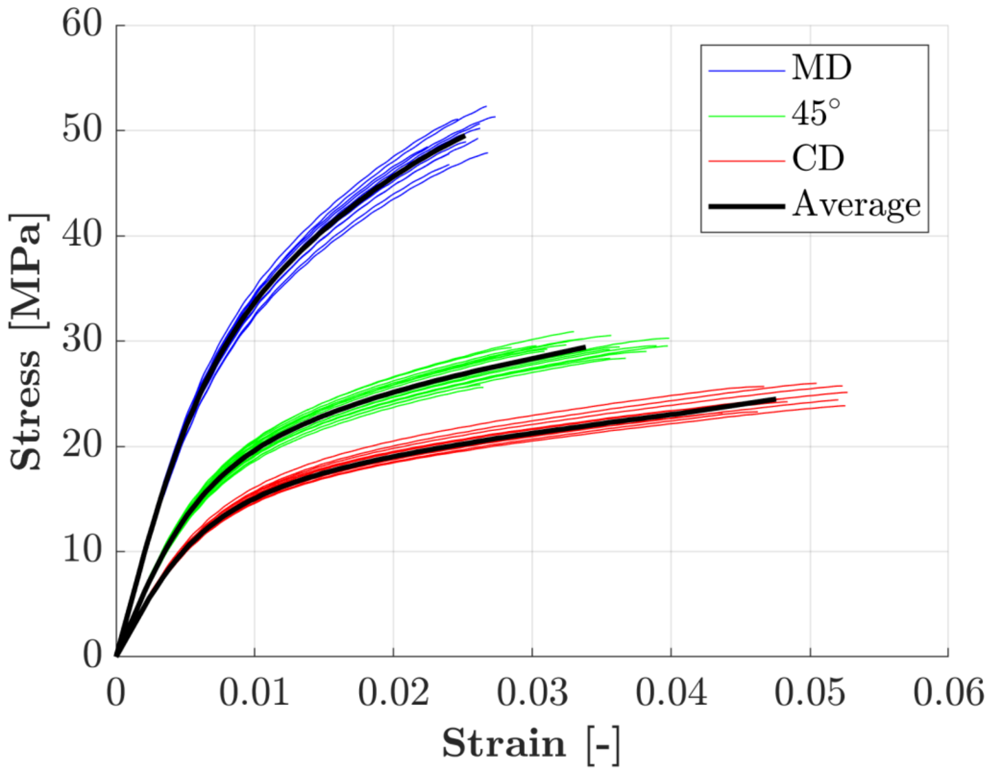
Material characterization

Uni-axial response of the material in MD, 45° and CD and the average response in each direction for characterization



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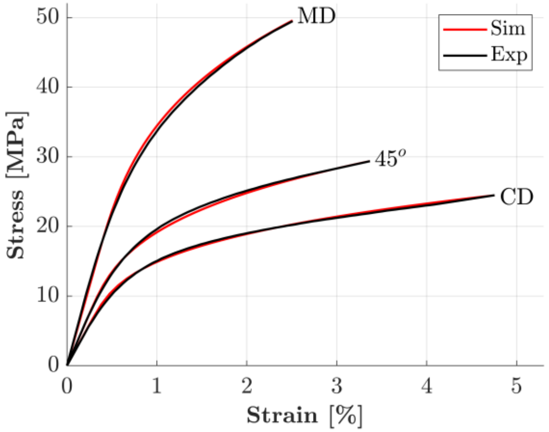
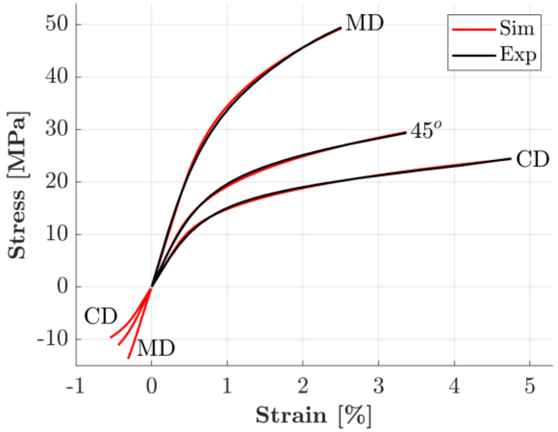
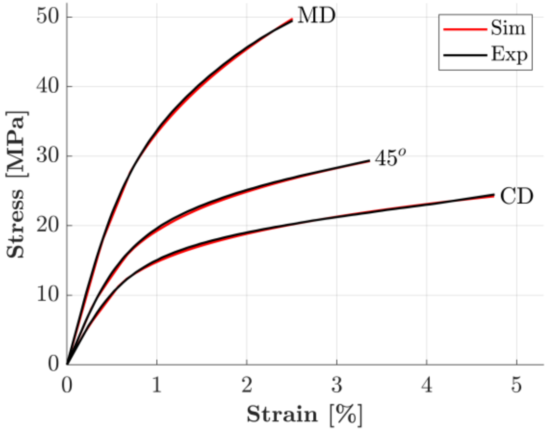
Finite element simulation



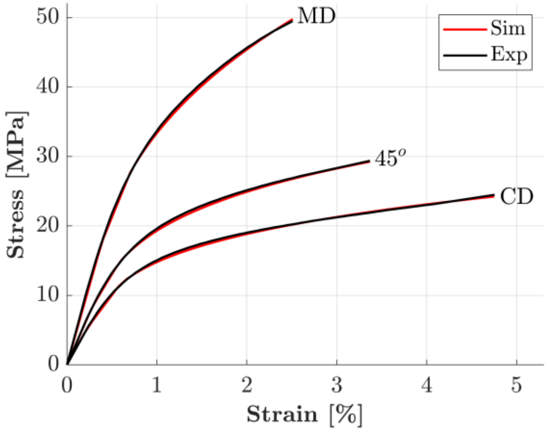
Hill calibration



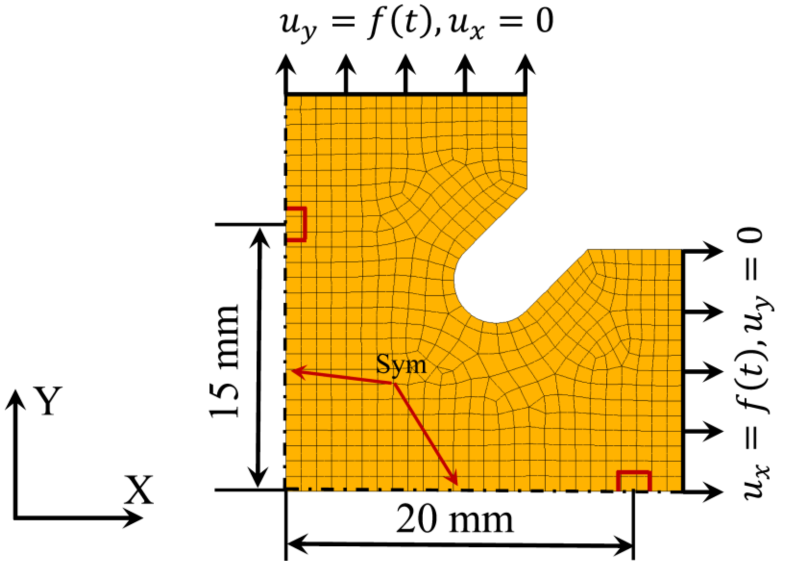
Hoffman calibration



Xia calibration for k = 1



Xia calibration for k = 2



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Analysis and Results

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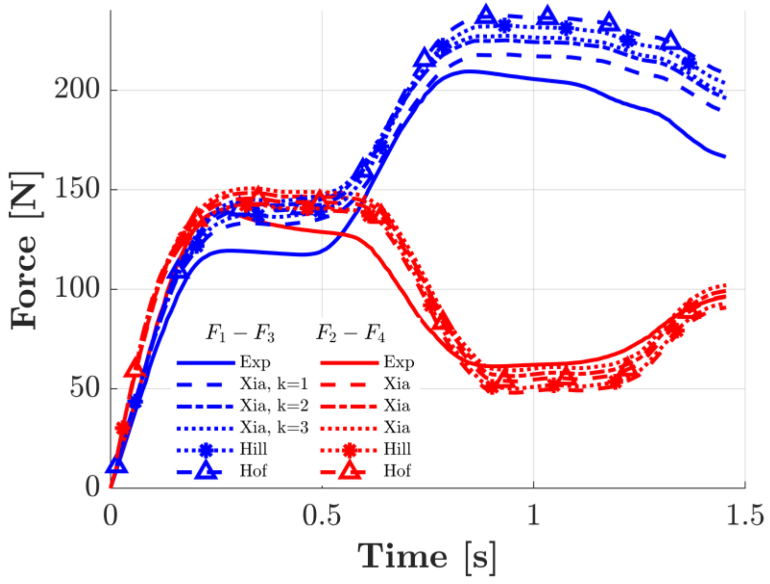
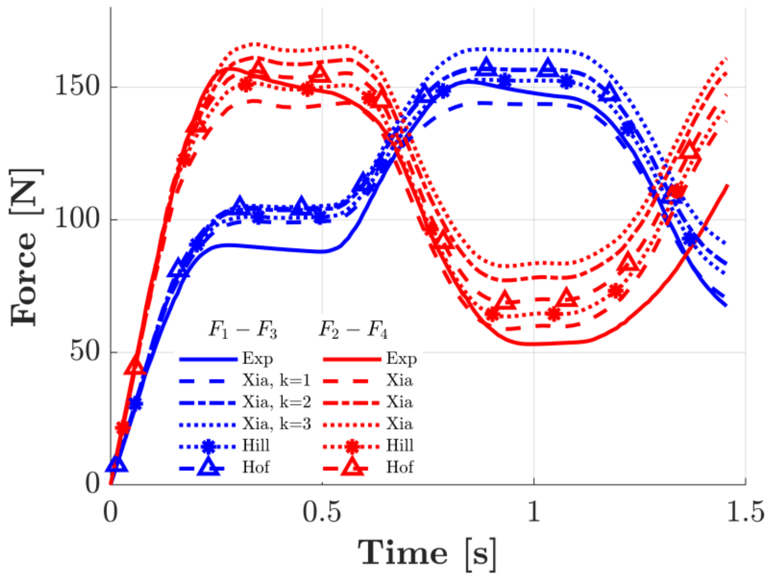
Comparison results

Comparison of experiment to the simulation results for the Hill, Hoffman, and Xia models

The Xia shape parameters k = 1, 2, and 3 are used in the simulation



MD-CD Test



45-Rot Test

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Comparison discussion

The simulated reaction forces using the Hill, Hoffman, and Xia model for $k \leq 2$ are in agreement with the experiment

Xia model with $k \geq 3$ consistently shows a stiffer response compared to the Hill and Hoffman

Xia model with shape parameter $k = 2$, the bi-axial response is similar to that from the Hoffman model

For the Xia and Hoffman, this stiffer response is due to k, and $(\boldsymbol{\sigma}^T \boldsymbol{q})$, respectively

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Comparison discussion

Xia with $k = 1$, followed by Hill, present the closest responses to the experiments

For symmetric tension-compression response, the Hill model is able to capture adequately the biaxial stresses

Featuring different tension-compression for Hoffman requires recalibration of the model

For Xia, presenting a different tension-compression doesn’t require recalibration (uncoupling of sub-surfaces)

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Summary of the comparison

	Hill	Hoffman	Xia
Convexity	Conditionally convex depending on the orthotropic plastic matrix	Conditionally convex depending on the orthotropic plastic matrix	Unconditionally convex
Number of plastic parameters	6	8	12

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User material subroutine and the fitting tool

The user material source codes for Hoffman and Xia will be shared as well as the Matlab calibration tool of Xia

```
% Xia fitting tool for 2-D plane case by Mossab Alzweighi
% KTH Royal Institute of Technology
% mossab@kth.se
%%
clc;
clear all;
close all;
set(groot,'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
%% load experimental data
ExpFile = importdata('FinalExp.txt'); % input data with the format: strain[%],
ExpFile(:,1) = ExpFile(:,1)/100; % convert strain from [%] to [-]
Rp = 0.02/100; % define plastic yield strain value

```

```
      subroutine usermatpsHof(
     &                 matId,  elemId, kDomIntPt,  kLayer,  kSectPt,
     &                 ldstep, isubst, keycut,
     &                 nDirect, nShear, ncomp, nStatev, nProp,
     &                 Time, dTime, Temp, dTemp,
     &                 stress, ustatev, dsdePl, sedEl, sedPl, epseq,
     &                 Strain, dStrain,  epsPl,  prop,  coords,
     &                 var0,  defGrad_t,  defGrad,
     &                 tsstif,  epsZZ,  cutFactor,
     &                 var1,  var2,  var3,  var4,  var5,
     &                 var6,  var7)
***********************************************************************
***********************************************************************
  --- Hoffman elastic-plastic model with anisotropic proprties and
c --- assymteric tension-compression response
c --- for 2-D plane stress case.
c --- Mossab Alzweighi
c --- KTH Royal Institute of Technology
c --- mossab@kth.se
```

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End

Thank you for your attention!

Mossab@kth.se

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